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Cyclic cosmology, conformal symmetry and the metastability of the Higgs

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ABSTRACT

Recent measurements at the LHC suggest that the current Higgs vacuum could be metastable with a modest barrier (height $(10^{10-12}$ GeV$)^4$) separating it from a ground state with negative vacuum density of order the Planck scale. We note that metastability is problematic for standard bang cosmology but is essential for cyclic cosmology in order to end one cycle, bounce, and begin the next. In this Letter, motivated by the approximate scaling symmetry of the standard model of particle physics and the primordial large-scale structure of the universe, we use our recent formulation of the Weyl-invariant version of the standard model coupled to gravity to track the evolution of the Higgs in a regularly bouncing cosmology. We find a band of solutions in which the Higgs field escapes from the metastable phase during each big crunch, passes through the bang into an expanding phase, and returns to the metastable vacuum, cycle after cycle after cycle. We show that, due to the effect of the Higgs, the infinitely cycling universe is geodesically complete, in contrast to inflation.

The recent discovery at the Large Hadron Collider of a Higgs-like particle with mass 125–126 GeV [1,2], combined with measurements of the top quark mass [3], implies that the electroweak Higgs vacuum may be metastable and only maintained by a modest energy barrier of height $(10^{10-12}$ GeV$)^4$ that is well below the Planck density. The conclusion [4] is based on computing the running of the standard model Higgs quartic coupling λ and finding that it switches from positive to negative when the expectation value of the Higgs field h exceeds 10^{10-12} GeV, under the strong assumption that there is no new physics at energies less than the Planck scale that significantly alters the predictions of the standard model. Since there is no evidence at present for disruptive new physics, we wish to take the metastability seriously and consider its cosmological implications. The Higgs effective potential is shown in Fig. 1 in a series of insets that show the potential on progressively smaller scales (energy density and field are expressed in Planck units).

One consequence is that the current phase of the universe, dominated by dark energy and characterized by accelerated expansion, has a finite lifetime before decaying to a contracting phase with large negative potential energy density. There are even more significant consequences for the early universe. The metastability of the Higgs causes serious problems for big bang inflationary cosmology [5]. If the Higgs is not trapped within the barrier when the universe first emerges from the big bang, it will rapidly evolve to a state with very negative potential energy density, causing the universe to contract. However, as is apparent from Fig. 1, the barrier height is so low, the true vacuum so negative (of order the Planck density), and the metastable field range so narrow compared to the Planck scale that the likelihood of being trapped is tiny. As for inflation [6], matters are worse. A generic problem for inflation is that it requires unlikely initial conditions in order to take hold: namely, the inflaton field must be smooth over more than a Hubble volume [7]. Now, in addition to the inflaton field being smooth, the Higgs must be trapped over that same volume or else its negative potential energy density will overwhelm the inflationary potential energy density, preventing inflation from occurring. A metastable Higgs thereby makes inflation more improbable. Even if inflation does begin, de Sitter fluctuations will tend to kick the Higgs field over the barrier if inflation begins at sufficiently high energies [5]. This effect can terminate inflation at any time, well before the last 60 efolds. In sum, the big bang inflationary picture combined with the metastable Higgs suggests our universe's past is unlikely and its future is precarious. The observed universe seems even more unlikely if the Higgs vacuum is part of a complex energy landscape that in some places includes stable Higgs vacua.

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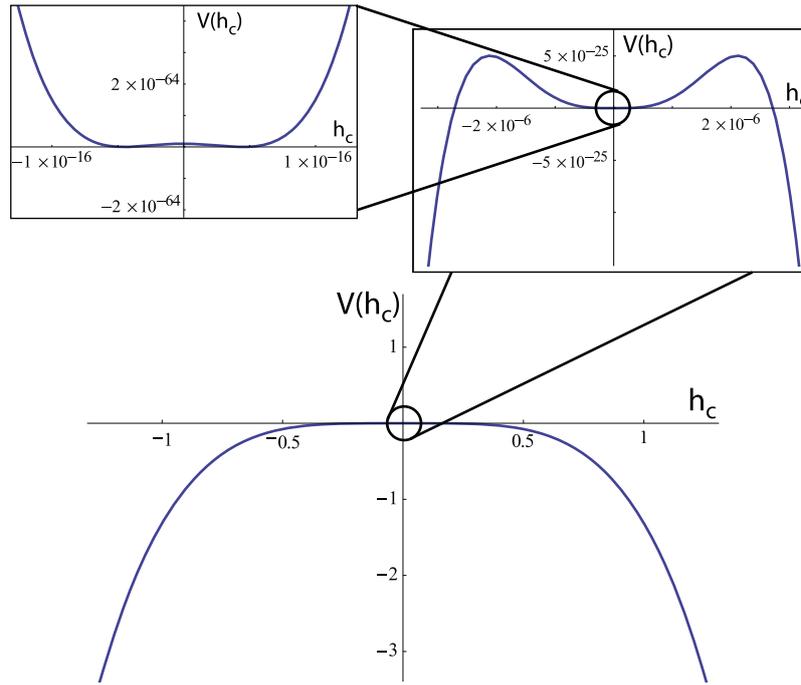
Fig. 1. A sequence of expanded views of the metastable Higgs potential for the standard model suggested by recent LHC data. The Higgs field $|h_c|$ and its potential energy density $V(h_c)$ values are expressed in Planck units. The bottom figure shows that most of the Higgs field range corresponds to large negative energy density. The effective potential for $|h_c| > 1$ is subject to quantum gravity corrections, so the shape beyond $|h_c| = 1$ is unknown. The middle inset (upper right) shows the energy barrier whose height is nearly 25 orders of magnitude below the Planck density. The final inset (upper left) shows the current (spontaneous symmetry breaking) vacuum.

By contrast, a metastable Higgs fits cyclic cosmology [8,9] to a tee. The current vacuum is *required* to be metastable (or long-lived unstable), according to the cyclic picture, in order for the current phase of accelerated expansion to end and for a big crunch/big bang transition to occur that enables a new cycle to begin [10]. So, it is essential that there exist scalar fields that can tunnel (or slowly roll) from the current vacuum with positive potential density to a phase where the potential energy density is negative and steeply decreasing as the magnitude of the field grows. The negative potential energy density triggers a reversal from expansion to contraction that continues as the field rolls downhill. For the cyclic model, this behavior would not only have to be part of our future, but also part of our past, describing the period leading up to the most recent bounce, a.k.a. the big bang.

Hence, a metastable Higgs could play an all-important role in cosmology that was not anticipated previously. To develop this idea, we constructed a theoretical formulation [11] that can incorporate all known physics and track the evolution of the Higgs through the big bounce. In constructing this formulation, we were guided by a basic principle that appears to pervade physics on the very smallest and very largest scales: scaling symmetry. On the micro-scale, the standard model of particle physics has a striking scaling symmetry if the small Higgs mass term (10^{-17} in Planck units) is omitted. This suggests that fundamental physics is conformally invariant and the desired mass term may emerge from the expectation value of another scalar field. On the cosmic scale, the Planck satellite [12] has shown the universe to be remarkably uniform and simple with nearly scale-invariant fluctuations on the largest observable scales. Together, these observations motivate us to consider Higgs models that incorporate scale symmetry from the start, including gravity: that is, Weyl-invariant actions that match phenomenology at the low energies probed by accelerators [11].

A key advantage of these theories for cosmology, as discussed in Refs. [11,13–15], is that they have classical solutions that make

it possible to trace their complete evolution through big crunch/big bang transitions. The completion introduces a period between big crunch and big bang during which, in the classical, low-energy description, the coefficient of the Ricci scalar in the gravitational action changes sign. This brief, intermediate 'antigravity' phase is somewhat analogous to the propagation of a virtual particle within a scattering amplitude describing incoming and outgoing on-shell particles. In our case, the incoming collapsing phase and the outgoing expanding phase both involve 'normal' Einstein gravity. We have shown that, in appropriate conformal gauges, the classical evolution across such a bounce is well-defined and essentially unique.

In this Letter, we explore the question of whether there exist cyclic solutions that will return the Higgs to its metastable vacuum after each big crunch/big bang transition. This is not obvious if the Higgs field in the current phase lies in a shallow potential well, separated by a small barrier (as compared to the Planck scale) from a very deep negative minimum of Planck scale depth [4]. One can imagine that the Higgs field would pop out of the metastable vacuum during the crunch and never find its way back again in the next cycle, in which case a metastable Higgs would be incompatible with a cyclic universe.

For our analysis, we use a Weyl-invariant action $S = \int d^4x \, \mathcal{L}(x)$ that describes gravity and the standard model

$$
\begin{aligned}
\mathcal{L}(x) = \sqrt{-g}\Bigg[& \frac{1}{12}\big(\phi^2 - 2H^\dagger H\big)R(g) \\
& + g^{\mu\nu}\left(\frac{1}{2}\partial_\mu\phi\partial_\nu\phi - D_\mu H^\dagger D_\nu H\right) \\
& - \left(\frac{\lambda}{4}\big(H^\dagger H - \omega^2\phi^2\big)^2 + \frac{\lambda'}{4}\phi^4\right) \\
& + L_{\mathrm{SM}}\big(\text{quarks, leptons, gauge bosons,} \\
& \quad \text{Yukawa couplings to } H, \text{ dark matter.}\big)\Bigg]
\end{aligned}
\tag{1}
$$

Here L_{SM} represents the standard model Lagrangian except for the kinetic and self interaction terms of the Higgs doublet H, which are explicitly written in the first three lines of Eq. (1). The additional scalar field ϕ is a singlet under $SU(2) \times U(1)$, and, therefore, it cannot couple to the standard model fields, except for the Higgs, as indicated on the third line, where ω is a small parameter (10^{-17} in Planck units) that determines the Higgs vacuum expectation value and the Higgs mass. Neutrino masses and simple models of the dark matter may be included through rather modest extensions involving gauge singlet fields. Both ϕ and H are conformally coupled scalars, with the special coefficient $1/12$ required by the local Weyl symmetry. The action is invariant under Weyl rescaling with an arbitrary local function $\Omega(x)$ as follows:

$$g_{\mu\nu} \to \Omega^{-2} g_{\mu\nu}, \qquad \phi \to \Omega\phi, \qquad H \to \Omega H,$$
$$\psi_{q,l} \to \Omega^{3/2} \psi_{q,l}, \qquad A_\mu^{\gamma,W,Z,g} \to \Omega^0 A_\mu^{\gamma,W,Z,g}. \qquad (2)$$

Any function that depends only on the ratio H/ϕ or $(\det(-g))^{1/8} H$, or $(\det(-g))^{1/8}\phi$, is Weyl-invariant.

Note the relative minus sign between ϕ and H kinetic energy terms and couplings to the Ricci scalar R. H is the physical scalar field corresponding to the Higgs, so in the low energy theory there is no choice about its canonically normalized kinetic energy term, and, then, conformal symmetry fixes its coupling to R. Then, the coupling to the Ricci scalar must be opposite for ϕ in order to obtain the proper positive overall coefficient of the Ricci scalar in Eq. (1); with this choice, its kinetic energy must be opposite as well to maintain conformal invariance. At first sight, ϕ appears to be a ghost. However, this is an illusion, as can be demonstrated by choosing a Weyl gauge $\Omega(x)$ where ϕ is constant throughout spacetime so that it is eliminated as a physical degree of freedom. In this gauge (referred to as c-gauge in [14]) $\phi(x) \to \phi_0$ we can express the physically important dimensionful parameters (the Newton constant G, the cosmological constant Λ, and the electroweak scale v) as:

$$\frac{1}{16\pi G} = \frac{\phi_0^2}{12}, \qquad \frac{\Lambda}{16\pi G} = \frac{1}{4}\lambda'\phi_0^4, \qquad H_0^\dagger H_0 = \omega^2 \phi_0^2 \equiv \frac{v^2}{2}. \quad (3)$$

The original action, Eq. (1), determines the conformally-invariant effective action for the relevant cosmological degrees of freedom for a homogeneous and isotropic Friedmann–Robertson–Walker (FRW) universe [15]:

$$\int d\tau \left(-\frac{1}{2e} \left[(\partial_\tau(a\phi))^2 + (\partial_\tau(ah))^2 \right] \right.$$
$$\left. - e \left[a^4 V(\phi, h) + \rho_r + C\sqrt{\rho_r} a^2 h^2 + \mathcal{K}(\phi^2 - h^2) a^2 \right] \right) \qquad (4)$$

where τ is conformal time, e is the lapse function, C is a dimensionless constant, $\mathcal{K}$ is the spatial curvature, and the homogeneous function of degree four, $V(t\phi, th) = t^4 V(\phi, h)$ describes the Higgs potential. Here we treat the gauge bosons and fermions as a radiation fluid at temperature T, inducing a term of the form $T^2 H^\dagger H \sim \sqrt{\rho_r} a^2 h^2$ in the effective potential for the Higgs field, where $\rho_r/a_E^4 \propto T^4$ is the radiation density in Einstein frame and ρ_r is a constant.

The classical equations following from Eq. (4) can be analyzed in various conformal gauges (c-gauge, E-gauge, γ-gauge) as described in Ref. [14]. In each gauge we label the fields with a corresponding subscript (a_c, ϕ_c, h_c) or (a_E, ϕ_E, h_E) or $(a_\gamma, \phi_\gamma, h_\gamma)$. In the c-gauge already described, the conformal gauge freedom in (2) is used to set $\phi_c = \phi_0 = 1$ in Planck units, eliminating the ϕ degree of freedom. In the Einstein gauge, the coefficient of the Ricci scalar in (1) is set to a constant $\frac{1}{12}(\phi_E^2 - h_E^2) = \frac{1}{2}$, reducing (ϕ, h)

to a single scalar degree of freedom. Finally, in the unimodular or γ-gauge the determinant of the metric is set equal to minus one, or $a_\gamma = 1$. In this gauge there clearly is no cosmological singularity, while all the dynamics including the expansion of the universe is represented smoothly by the fields ϕ_γ and h_γ. The cosmological evolution may be studied in any gauge, but for the purposes of analyzing and interpreting the solutions it is often useful to translate the results into gauge-invariant quantities whose physical meaning is clear in some particular gauge. One such quantity is $h/\phi = h_c/1 = h_E/\phi_E = h_\gamma/\phi_\gamma$, which represents the magnitude of the Higgs field in Planck units in c-gauge (h_c). Another is [14] $\chi = \frac{1}{6}(-g)^{\frac{1}{4}}(\phi^2 - h^2) = \frac{1}{6}(-g_c)^{\frac{1}{4}}(1 - h_c^2) = \frac{1}{6}(\phi_\gamma^2 - h_\gamma^2) = a_E^2 \,\mathrm{sign}(\chi)$, which represents the square of the scale factor in E-gauge, $a_E^2 = |\chi| = \frac{1}{6}|\phi_\gamma^2 - h_\gamma^2|$; note that $\mathrm{sign}(\chi)$ is gauge invariant. Yet a third useful gauge-invariant quantity is $a\phi = a_c \cdot 1 = 1 \cdot \phi_\gamma$.

For finding and exploring bouncing FRW cosmologies, the unimodular γ-gauge is most convenient, as discussed in Ref. [14]. In the case that the Higgs potential is of purely quartic form, $V(\phi_\gamma, h_\gamma) = \frac{1}{4}(\lambda h_\gamma^4 + \lambda'\phi_\gamma^4)$, we have produced complete analytic solutions [15] for $(\phi_\gamma(\tau), h_\gamma(\tau))$ for all values and signs of (λ, λ'), including radiation ρ_r, curvature $\mathcal{K}$, and all initial conditions for the fields (ϕ_γ, h_γ) and their derivatives $(\dot\phi_\gamma, \dot h_\gamma)$. (We did not consider the thermal contribution proportional to C in Eq. (4) in [15], but its inclusion is trivial in the same approach.) These studies yielded all solutions not just some special cases, thus teaching us how to construct bouncing cosmological spacetimes for all the fields $\phi, h, g_{\mu\nu}$.

The realistic Higgs potential analyzed in this Letter is a small deformation of the quartic potential above for which exact analytic solutions were obtained, so the generic properties of the cosmological solutions are similar. We will discuss the realistic case below using numerical methods but guided by our knowledge of the exact solutions so that we know our solutions are generic rather than based on wishful assumptions about initial conditions.

The evolution of an FRW universe with Higgs field and radiation is represented in γ-gauge by the two dynamical quantities ϕ_γ and h_γ. Using the gauge invariant quantity $|\chi| = a_E^2 = |\phi_\gamma^2 - h_\gamma^2|$, one sees that the cosmic singularity in Einstein frame at $a_E = 0$ corresponds to crossing the light-cone in the ϕ_γ–h_γ plane (see [14,15]). In unimodular γ-gauge, for which $a_\gamma = 1$, solutions are smooth across the light-cone and, hence, can be continued through the big crunch/big bang transition. Thus, in this context, unimodular gauge and all gauges smoothly related to it are regarded as good conformal gauges. In contrast, in E-gauge some quantities are singular at $a_E = 0$. For example, E-gauge assumes that the gauge-invariant quantity $1 - h^2/\phi^2$ is non-negative; however, our complete set of solutions show that this is not true for generic initial conditions. Hence, E-gauge is a bad gauge choice for studying the complete evolution of FRW cosmologies.

To study the metastable Higgs, we numerically solve the equations of motion for the action in Eq. (4) using the quantum corrected Higgs potential:

$$V(\phi, h) \equiv \frac{1}{4}\phi^4 \left(\lambda' + \lambda(h/\phi)\left(\frac{h^2}{\phi^2} - \omega^2\right)^2 \right), \qquad (5)$$

where the factor multiplying ϕ^4 is Weyl-invariant. In the c-gauge, with $\phi_c = \phi_0 = 1$ in Planck units, this looks like the familiar Higgs potential including the quantum corrected running coupling $\lambda(h/\phi_0)$. Then $\omega = (246\ \text{GeV})/\phi_0$ gives the Higgs expectation value, and $\frac{1}{4}\lambda'\phi_0^4$ gives the cosmological constant, both in Planck units in today's Higgs vacuum.

In order to study cyclic solutions, in this Letter we shall artificially take λ' to be negative and small compared to all other

scales. This is to mimic an additional effect needed in a cyclic model, where λ' would be replaced by a field that rolls or tunnels from small positive energy density (corresponding to the current dark energy density) to a negative value to trigger the transformation from expansion to contraction. This field could even be the Higgs if tunneling is included. That is, when the Higgs tunnels, it jumps to a state with negative potential energy density that plays the same role as λ' in transforming the universe from expansion to contraction. For the purposes of exhibiting the stable cyclical behavior of the Higgs, however, the same effect can be obtained by setting $\lambda' < 0$, in which case the transformation occurs when the density of matter and radiation in the current spontaneous symmetry breaking vacuum falls below $|\lambda'|$.

For the running quartic coupling $\lambda(h/\phi)$, we assume the form computed in Ref. [4]. Rather than use the precise result, which cannot be easily expressed in closed form, we use a simplified expression that captures the essential features: a metastable, spontaneous symmetry breaking Higgs vacuum, with a barrier of $\sim (10^{10-12}\text{ GeV})^4$ separating it from the true negative energy density vacuum. A simple parameterization that reproduces the key features in Fig. 1 is:

$$\lambda(h/\phi) = \lambda_0\left(1 - \epsilon\ln\left(\frac{h}{\omega\phi}\right)^2\right) \tag{6}$$

where λ_0 is chosen to fit the observed Higgs mass in today's Higgs vacuum at $h/\phi = \omega \approx 10^{-17}$, and ϵ is chosen such that the quartic coupling passes below zero at $h_c \approx 10^{12}$ GeV. (To avoid logarithmic singular behavior for our numerical computations, we include small cutoff parameters inside the log not shown here because the solutions are insensitive to them.)

Our principal finding is that there exists a continuous band of solutions that undergo repeated cycles of expansion, contraction, crunch, bang and back to expansion again in which the Higgs field returns to the metastable Higgs vacuum during each expansion phase and that these infinitely cycling solutions are geodesically complete. The band corresponds to solutions whose total Higgs kinetic plus potential energy density lies in a range that extends from a little above the barriers in Fig. 1 (second inset) to the local minimum of the potential corresponding to the current vacuum. As long as the Higgs initial condition lies in this band after the bang, it returns to the stable band after each subsequent big bang. It is then trapped within the depression within the potential barriers (second inset of Fig. 1) and its kinetic energy red shifts until it settles into a spontaneous symmetry breaking vacuum (third inset of Fig. 1). Due to the negative cosmological constant (λ') the total energy density eventually becomes negative and the evolution reverses from expansion to contraction. Now the Higgs field kinetic energy density begins to blue shift until its oscillations grow to the point where it jumps beyond the barriers and approaches the Planck scale at the big crunch. After passing through the region with $h_\gamma^2 > \phi_\gamma^2$, the process begins again.

The trajectory in the ϕ_γ–h_γ plane is illustrated in Fig. 2 for the case of no anisotropy. The evolution of ϕ, h and the gauge-invariant ratio $h_c = h/\phi$ corresponding to the Higgs field value in c-gauge are shown in Fig. 3. The 45 degree lines correspond to $a_E^2 = |(1/6)(\phi_\gamma^2 - h_\gamma^2)| = 0$, a singularity corresponding to either a big crunch or a big bang. Between the crunch and bang is a brief intervening period in which $h_\gamma^2 > \phi_\gamma^2$ and the coefficient of R in the action (1) changes sign, as discussed in Ref. [14]. Fig. 2 shows that the solution passes without incident through each crunch/bang transition. As the trajectory passes through the light-cone-like boundaries in Fig. 2, $|h_\gamma/\phi_\gamma|$ approaches unity (see the jumps in Fig. 3a), so the Higgs field has popped out of the metastable vacuum, as anticipated. Then, beginning from the left
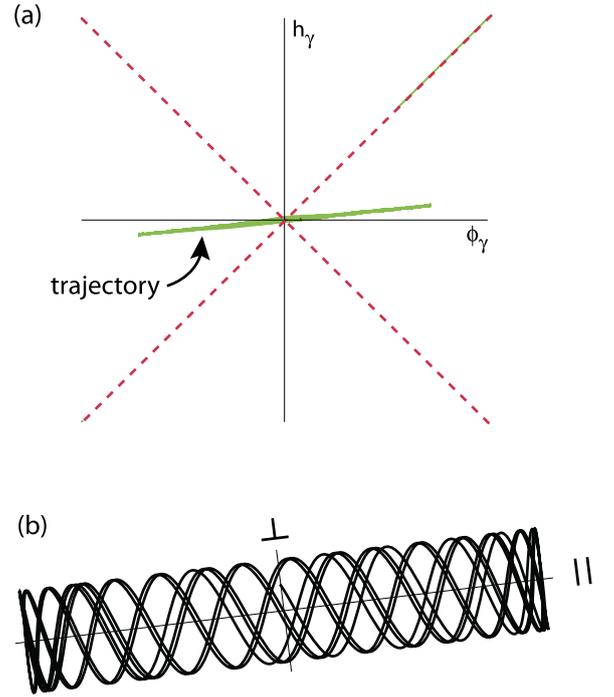
Fig. 2. Figure (a) shows the back-and-forth trajectory of h_γ and ϕ_γ in the ϕ_γ–h_γ plane. For the purposes of illustration, the plot is in Planck units and an unrealistic Higgs mass of order unity (in Planck units) has been chosen to make the features visible. For smaller Higgs masses, the behavior is qualitatively similar. Although the trajectory appears to be a straight slanted line when shown on this scale, figure (b) is a blow-up that shows the trajectory to be more complicated. In this plane, the light-cone-like lines correspond to the singularity in the Einstein frame, $a_E^2 = (1/6)|\phi_\gamma^2 - h_\gamma^2| = 0$. In the left and right quadrants, the trajectory moves along an approximately fixed angle corresponding to a fixed Higgs value (constant $h_c = h_\gamma/\phi_\gamma$), the current vacuum. As the trajectory moves towards the light-cone, the universe contracts (a_E shrinks); crossing the light-cone corresponds to the big crunch/big bang transition; and moving away from the light-cone corresponds to expansion (growing a_E). In figure (b), the axis perpendicular to the trajectory ($\perp$) has been greatly expanded to show the combination of rapid, small amplitude oscillations of the Higgs around the vacuum ($\perp$ direction) and the comparatively slow cycles of expansion and contraction of the scale factor ($\parallel$ direction). From this figure it can be seen that cycles are quasi-periodic: similar but not identical, they explore an invariant torus in phase space as expected from the KAM theorem.

quadrant say, the trajectory goes through a period of contraction, passes through a crunch (the first 45 degree line) and bang (the second 45 degree line), and enters a period of expansion where it traverses deep into the right quadrant, corresponding to increasing $a_E^2 \sim \chi \propto \phi_\gamma^2 - h_\gamma^2$. During this phase, the Higgs field is observed to move towards zero and, as the expansion continues, to oscillate and slowly settle down (due to Hubble red shift) into one of the symmetry breaking vacua, as discussed above. Due to red shifting, the sum of the (positive) radiation and Higgs oscillatory energy densities plus the (negative) cosmological constant term $\lambda'\phi_\gamma^4$ eventually reaches zero. The Hubble expansion reverses to contraction, the trajectory begins to move towards the left quadrant, the radiation and Higgs oscillation densities begin to grow due to blue shifting until the crunch and bounce, and the cycle begins again. The cycles are not identical, as can be seen from the back and forth trajectories over several cycles in Fig. 2b and by carefully comparing the Higgs oscillations from one cycle to the next in Fig. 3a. The classical equations turn out to obey the assumptions of the Kolmogorov–Arnold–Moser (KAM) theorem, according to which a weakly nonlinear perturbation of a classically integrable system, generically deforms but does not remove the invariant tori in phase space. In our case, the equations are integrable in the case of no coupling between h and ϕ, as shown
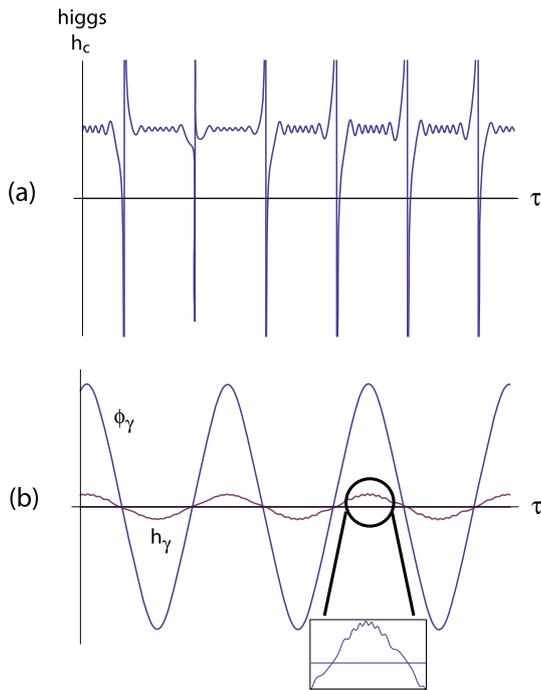
Fig. 3. Graphs of the evolution of (a) the Higgs field ($h_c = h_\gamma/\phi_\gamma$), and (b) the scale factor ($a_c = \phi_\gamma$) and h_γ, for the trajectory shown in Fig. 2. For the purposes of illustration, the plot is in Planck units and an unrealistic Higgs mass of order unity (in Planck units) has been chosen to make the features visible. For smaller Higgs masses, the behavior is qualitatively similar. In (a), the Higgs oscillates rapidly with small amplitude around a fixed value (the current vacuum) for most of a cycle and then spikes to large values up to the Planck scale during the crunch/bang transition. When the spikes in (a) cross ± 1 (the Planck scale, beyond the range shown here), the light-cone in Fig. 2a is being crossed, corresponding to a big crunch/big bang transition where a_E vanishes.

Fig. 4. Plot of the Einstein frame scale factor a_E and Higgs field value in γ-gauge h_γ versus cumulative FRW time, $t = \int d\tau\, a_E(\tau)$. If the entropy increases by a constant factor every cycle (*e.g.,* due to the production of radiation that couples to the Higgs when the Higgs goes through rapid variation during the crunch/bang transition), then so does the scale factor (solid curve) leading to exponential growth over many cycles, as indicated by the slanted line. The Einstein-frame temperature at given cosmic time t is the same as it was a cycle earlier or will be a cycle later; and the behavior of $|h_\gamma| a_E^{1/2}$ (shown in red dashed curve), is the same on average from cycle to cycle. That is, as the maximum $a_E^2 = \phi_\gamma^2 - h_\gamma^2$ increases from one cycle to the next, the amplitude of the oscillations in $|h_c| = |h_\gamma/\phi_\gamma|$ decreases.

in [15], and the small coupling between h and ϕ is a perturbation. Hence, the system cycles forever in quasi-periodic fashion and only explores a torus in phase space that is stable under perturbations, as illustrated in Fig. 2b.

In Fig. 3b, ϕ_γ oscillates smoothly while the evolution of h_γ has barely detectable, high frequency oscillations (magnified in the inset) corresponding to the Higgs field oscillating back and forth in the potential well of the metastable phase.

Although the figures illustrate the case with no anisotropy, the effects of anisotropy (combined with radiation) can be easily surmised based on the results in Ref. [14]. Namely, anisotropy only has a significant role near the big crunch/big bang transition, modifying all passes through the light-cone shown in Fig. 2a. The paths are distorted so that they travel precisely through the origin of the ϕ_γ–h_γ plane, traverse a small loop in the upper or lower quadrants and then exit through the origin again into the next big bang expansion phase. The size of the loop depends on the radiation density; higher radiation density results in a smaller loop [14]. (To compare cases with and without anisotropy, see Fig. 1 in [14].) This imposes a significant dynamical behavior on (h_γ, ϕ_γ) where they are both forced to be zero at the bang, but approach a ratio which is exactly 1, namely, $h_c = h_\gamma/\phi_\gamma = 1$ at the crunch/bang [14]. After the bang and far from the singularity, the distorted path approaches close to the trajectory without anisotropy. So anisotropy only helps ensure that the Higgs field is trapped in the next phase of expansion.

It is natural to imagine that, during the bounce, significant radiation density (parameterized by ρ_r) is produced by degrees of freedom that couple to the Higgs because the Higgs undergoes such rapid change. In Ref. [14], it was shown that radiation pro-

duced during the antigravity phase backreacts by speeding up ϕ_γ and h_γ such that the universe emerges from the big bang more rapidly than it entered the big crunch, with the consequence that the scale factor a_E grows from cycle to cycle, as shown in Fig. 4. The duration of each cycle in proper FRW time is set by the value of the (negative) cosmological constant λ', so it does not change. Likewise, the temperature of the universe in Einstein frame at maximum scale factor does not change from cycle to cycle. This echoes the scenario envisaged in the cyclic model in Ref. [8], but, remarkably, here it is accomplished through the standard model with a metastable Higgs. What does change with each cycle is the amplitude of the oscillations in the Higgs field around the symmetry-breaking minimum. As radiation is produced, the parameter ρ_r grows, and the Higgs oscillation amplitude decreases.

Finally, we wish to demonstrate the surprising effect the Higgs can have in enabling cyclic universes to be geodesically complete, in contrast to inflationary scenarios, including "eternal inflation". Inflationary scenarios are well known to be at most semi-eternal to the future and, hence, necessarily dependent upon some assumed initial condition [16]. For our cyclic solutions, we have already emphasized in [14] and here that the solutions may be continued arbitrarily far backward in conformal time through bounce after bounce. We shall now compare cyclic and inflationary scenarios using a coordinate-invariant definition of geodesic completeness.

Inflationary scenarios, if they allow for *any* forms of energy other than inflationary potential energy at early times, generically lead to a singularity at finite conformal time in the past. In considering the global structure of inflationary spacetime, it is sometimes considered that the universe somehow became stuck an arbitrarily long time in the past in a positive energy false vacuum. In the flat slicing of pure de Sitter spacetime, the conformal time during inflation then stretches arbitrarily far back into the past. However, this superficial appearance of completeness is in fact a coordinate artifact. This may be seen by changing to the closed slicing of de Sitter spacetime. In this slicing, the universe 'bounces' at some finite conformal time in the past, so that inflation is in fact pre-

ceded by a deflationary phase in which all of inflation's successes during the expanding de Sitter phase would be precisely undone in the preceding, collapsing de Sitter phase. So in order to build a successful inflationary scenario, one must simply ignore the earlier collapsing phase and assume that the universe just somehow started out in the expanding phase. This is part of the well-known initial conditions problem of inflation.

Even without changing to the closed slicing, one can identify the problem by using the following coordinate-invariant definition of geodesic completeness: that generic time-like geodesics – the worldlines of massive freely falling particles – may be extended arbitrarily far backward in proper time. It is natural to measure the time along the particle worldline t_p in units of m^{-1} where m is the particle mass, *i.e.*, the magnitude of the action for the particle:

$$|\mathcal{S}| = \int m \, dt_p = \int d\tau \, \frac{m^2 a_E(\tau)^2}{\sqrt{p^2 + m^2 a_E(\tau)^2}} \tag{7}$$

(see, *e.g.*, [13]), where p is the particle's (conserved) canonical momentum (and τ is the conformal time, as before). In the case of an expanding de Sitter epoch stretching all the way back to zero scale factor, the best possible case for inflation, we have $a_E \propto -1/\tau$, where $\tau = -\infty$ corresponds to $a_E = 0$. The integral converges at the lower limit, meaning that the total proper time experienced by the particle is finite, so the spacetime is geodesically incomplete. Since this was the best possible case, it follows that, in the absence of an account of what preceded inflation, all inflationary scenarios, even 'eternal inflation' scenarios, are geodesically incomplete [16].

In contrast, with the same criterion, all of our Higgs cyclic models are geodesically complete. All massive particles receive a mass contribution from the Higgs, and so the quantity $m a_E$ in (7) should be replaced by $g h_E a_E$, where g is some coupling constant. Since ha is gauge-invariant, we can replace this $g h_E a_E = g h_\gamma a_\gamma = g h_\gamma$, where h_γ is the Higgs expectation value in unimodular gauge. As we have seen from our solutions in Fig. 3b, if no radiation is generated, the quantity h_γ oscillates at fixed amplitude for cycle upon cycle, and so the integral (7) diverges as τ is extended back into the past. If radiation *is* generated with each new cycle, the amplitude of h_γ increases as we follow the universe back into the past, as indicated in Fig. 4, and the argument becomes even stronger. There are two possibilities: either h_γ remains finite, or it diverges at some finite value of conformal time, τ_*. From the action (4) one sees that it will do so like $h_\gamma \propto 1/(\tau - \tau_*)$. In this case, the action for a massive particle in Eq. (7) will diverge at τ_*. Thus, with this definition of geodesic completeness, we conclude that all cyclic Higgs scenarios are geodesically complete to the past, whereas all inflationary scenarios are not.

In sum, we have shown that the Higgs, if it is metastable, has profound implications for cosmology. For big bang inflationary cosmology, metastability is problematic: it is unexpected, requires highly improbable initial conditions, and predicts a dire future in which the metastable phase ends and the universe collapses in a big crunch. By contrast, metastability dovetails with the cyclic picture for which decay of the current vacuum is a fundamental prediction, required to end the current cycle and begin the next. Remarkably, our Weyl-invariant formulation of the standard model has made it possible to construct an action that incorporates all known microphysics and, at the same time, has classical solutions that completely describe cyclic evolution, from bounce to expansion to contraction to bounce again, with each cycle reproducing similar physics that is like what we observe. We have further demonstrated how the Higgs naturally makes the cyclic scenario geodesically complete.

Acknowledgements

References

[1] S. Chatrchyan, et al., CMS Collaboration, Phys. Lett. B 716 (2012) 30, arXiv:1207.7235 [hep-ex].
[2] G. Aad, et al., ATLAS Collaboration, Phys. Lett. B 716 (2012) 1, arXiv:1207.7214 [hep-ex].
[3] CMS Collaboration, CMS-PAS-TOP-11-018 and CMS-PAS-TOP-11-015.
[4] G. Degrassi, S. Di Vita, J. Elias-Miro, J.R. Espinosa, G.F. Giudice, G. Isidori, A. Strumia, J. High Energy Phys. 1208 (2012) 098, arXiv:1205.6497 [hep-ph].
[5] J. Espinosa, G. Giudice, A. Riotto, J. Cosmol. Astropart. Phys. 0805 (2008) 002, arXiv:0710.2484.
[6] A.H. Guth, Phys. Rev. D 23 (1981) 347;
A.D. Linde, Phys. Lett. B 108 (1982) 389;
A. Albrecht, P.J. Steinhardt, Phys. Rev. Lett. 48 (1982) 1220.
[7] A.R. Liddle, D. Lyth, Cosmological Inflation and Large-Scale Structure, Cambridge University Press, Cambridge, 2000.
[8] P.J. Steinhardt, N. Turok, Science 296 (2002) 1436.
[9] P.J. Steinhardt, N. Turok, Phys. Rev. D 65 (2002) 126003.
[10] J.-L. Lehners, P. Mcfadden, P.J. Steinhardt, N. Turok, Phys. Rev. D 76 (2007) 103501.
[11] I. Bars, P.J. Steinhardt, N. Turok, Local conformal symmetry in physics and cosmology, arXiv:1307.1848, 2013.
[12] P. Ade, et al., Planck Collaboration, arXiv:1303.5082.
[13] I. Bars, S.-H. Chen, N. Turok, Phys. Rev. D 84 (2011) 083513, arXiv:1105.3606 [hep-th].
[14] I. Bars, S.-H. Chen, P.J. Steinhardt, N. Turok, Phys. Lett. B 715 (2012) 278, arXiv:1112.2470 [hep-th].
[15] I. Bars, S.-H. Chen, P.J. Steinhardt, N. Turok, Phys. Rev. D 86 (2012) 083542, arXiv:1207.1940 [hep-th].
[16] A. Borde, A.H. Guth, A. Vilenkin, Phys. Rev. Lett. 90 (2003) 151301, arXiv:gr-qc/0110012.